Estimating the competitive storage model with stochastic trend: A particle MCMC approach

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EcoSta 2019
National Chung Hsing University
June 25th 2019

The competitive storage model

The model [Deaton and Laroque, 1992] assumes:

- IID shocks (z_t) - supply/harvest
- Costly storage: $\beta = (1-\delta)/(1+r) < 1$
  - δ is the commodity depreciation rate and r is the interest rate
- Storage is non-negative
- A deterministic demand function, given as a function of a price: $D(p_t)$
- There exists an inverse demand function $P(x_t)$: $\quad D\left(P(x_t)\right) = x_t$
- The price is considered fixed when making storage decisions
- Speculators are assumed to hold rational expectations

Let I_t be the inventory level at time t. The amount of stocks at hand is then given by $x_t = (1-\delta)I_{t-1} + z_t$

The optimal storage policy implies $p_t = \max\left[P(x_t), \beta E_t p_{t+1}\right]$

The competitive storage model, continued

- The optimal storage policy implies $p_t = \max\left[P(x_t), \beta E_t p_{t+1}\right]$
- In equilibrium, supply must equal demand, leading to the following price function:

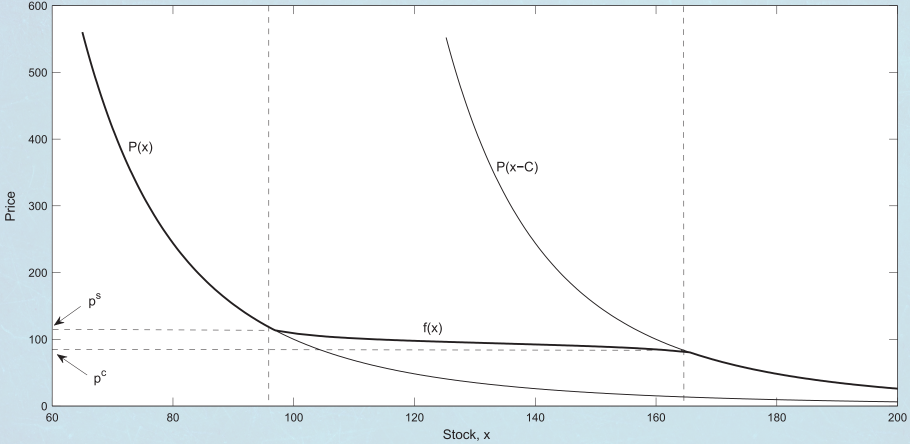
$$\begin{aligned} f(x) &= \max\left\{P(x), \bar{f}(x)\right\}, \\ \bar{f}(x) &= \beta E f\left((1-\delta)\sigma(x) + z\right), \\ \sigma(x) &= x - D(f(x)). \end{aligned} \tag{1}$$

- Following [Oglend and Kleppe, 2017], we assume storage is non-negative and **bounded from above** at $C \geq 0$:

$$f(x) = \min\left\{P(x-C), \max\left\{P(x), \bar{f}(x)\right\}\right\} \tag{2}$$

Equilibrium prices when storage is completely bounded

$$f(x) = \min\left\{P(x - C), \max\left\{P(x), \bar{f}(x)\right\}\right\}$$



Numerical solution

- We solve for $\sigma(x)$ and recover $f(x)$:

$$f_{\mathcal{S}}(x) = P\left(x - \sigma(x)\right)$$

$$\sigma(x) \approx \begin{cases} 0 & \text{if } x < \hat{x}^* \\ s(x) & \text{if } \hat{x}^* \leq x \leq \hat{x}^{**} \\ C & \text{if } x > \hat{x}^{**} \end{cases},$$

Iteratively, using initial values $\hat{x}^* = 0$, $\hat{x}^{**} = C$, $s(x)$ linear:

- $\hat{x}^*_{n+1} = D\left(\beta \int f_{\mathcal{S}}(z)\phi(z)dz\right)$
- $\hat{x}^{**}_{n+1} = D\left(\beta \int f_{\mathcal{S}}((1-\delta)C + z)\phi(z)dz\right) + C$
- Define the grid x_g as $[\hat{x}^*_{n+1}, \hat{x}^{**}_{n+1}]$
- For each grid point j, find updated $s(x)$ to be the solution in s to $s = x_g^{(j)} - D\left(\beta \int f_{\mathcal{S}}((1-\delta)s + z)\phi(z)dz\right)$

Stochastic trend

Expressing the storage model as a time series model for (observed) log-prices p_t:

$$\begin{aligned} p_t &= \log f(x_t), \\ x_t &= (1-\delta)\sigma(x_{t-1}) + z_t, \qquad z_t \sim \text{iid } N(0,1), \end{aligned} \tag{3}$$

Adding a stochastic trend:

$$\begin{aligned} p_t &= k_t + \log f(x_t), \\ k_t &= k_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \text{iid } N(0, v^2), \\ x_t &= (1-\delta)\sigma(x_{t-1}) + z_t, \qquad z_t \sim \text{iid } N(0,1), \end{aligned} \tag{4}$$

The inverse demand function is set to $P(x) = \exp(-bx)$

Objective: For given price data, estimate the storage model's structural parameters $\boldsymbol{\theta} = (v, \delta, b)$, together with the latent parameters (**k** and **x**)

Implicit stochastic trend

$$
\begin{aligned}
p_t &= k_t + \log f(x_t), \\
k_t &= k_{t-1} + \varepsilon_t, \qquad && \varepsilon_t \sim \text{iid } N(0, v^2), \\
x_t &= (1-\delta)\sigma(x_{t-1}) + z_t, \qquad && z_t \sim \text{iid } N(0, 1),
\end{aligned}
$$

For computational convenience, it is possible to express the stochastic trend implicitly, as $k_{t-1} = p_{t-1} - \log f(x_{t-1})$, and thus

$$
\begin{aligned}
p_t &= p_{t-1} + \log\left(\frac{f(x_t)}{f(x_{t-1})}\right) + \epsilon_t, \qquad && \epsilon_t \sim \text{iid } N(0, v^2), \\
x_t &= (1-\delta)\sigma(x_{t-1}) + z_t, && z_t \sim \text{iid } N(0, 1).
\end{aligned}
$$

Particle filter

The joint conditional probability density of p_t and x_t can be derived analytically:

$$p(p_t, x_t | p_{t-1}, x_{t-1}) \propto \frac{1}{v} \exp \Big[-\frac{1}{2v^2} \Big(p_t - p_{t-1} - \log f(x_t) + \log f(x_{t-1}) \Big)^2 - \frac{1}{2} \left(x_t - (1-\delta)\sigma(x_{t-1}) \right)^2 \Big]$$

- We estimate the marginal likelihood using the sampling importance resampling (SIR) particle filter [Gordon et al., 1993]

Particle marginal Metropolis-Hastings

$$
\begin{aligned}
p_t &= p_{t-1} + \log\left(\frac{f(x_t)}{f(x_{t-1})}\right) + \epsilon_t, \qquad && \epsilon_t \sim \text{iid } N(0, v^2), \\
x_t &= (1-\delta)\sigma(x_{t-1}) + z_t, && z_t \sim \text{iid } N(0, 1).
\end{aligned}
$$

Priors:
$v^2 \sim 0.1/\chi^2_{(10)}$, $\delta \sim \mathcal{B}(2, 20)$, $b \sim \mathcal{N}(0, 1)$

PMMH acceptance probability [Andrieu et al., 2010]:

$$
\min\left\{1, \frac{\hat{p}(y_{1:T}|\boldsymbol{\theta}_*)p(\boldsymbol{\theta}_*)}{\hat{p}(y_{1:T}|\boldsymbol{\theta}_{i-1})p(\boldsymbol{\theta}_{i-1})}\frac{q(\boldsymbol{\theta}_{i-1}|\boldsymbol{\theta}_*)}{q(\boldsymbol{\theta}_*|\boldsymbol{\theta}_{i-1})}\right\} \tag{5}
$$

We use a symmetric proposal density $q(\boldsymbol{\theta}_{i-1}) \sim N(\boldsymbol{\theta}_{t-1}, \Sigma)$, which entails that Eq. (5) is not dependent on q.

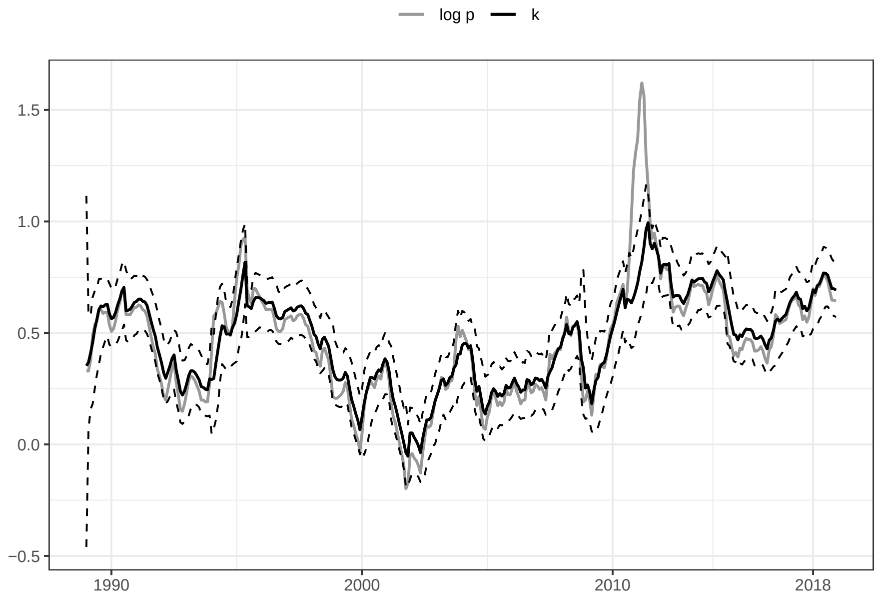
Σ is set adaptively [Haario et al., 2001].

Application

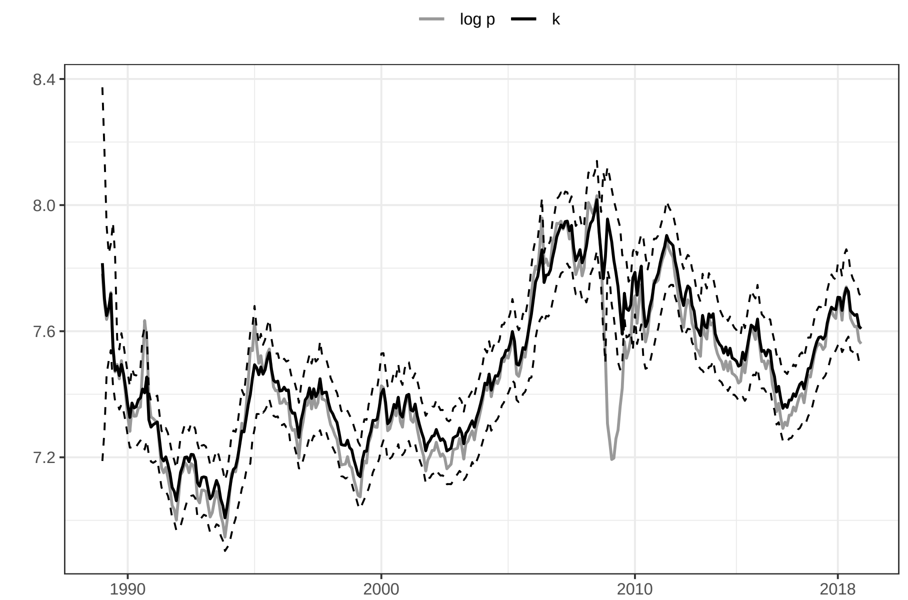
- The estimation methodology is applied to monthly commodity prices
- $r = 1.05^{1/12} - 1,\ C = 10$
- Importance density: $q_t(x_t, x_{t-1}) \sim N\left((1-\delta)\sigma(x_{t-1}), 1\right)$.

		natgas	coffee	cotton	aluminium
	Acc. rate	0.35	0.24	0.35	0.37
v	Post. mean	0.097	0.061	0.046	0.045
	Post. std.	0.008	0.004	0.003	0.002
	ESS	566	604	792	843
δ	Post. mean	0.012	0.002	0.001	0.001
	Post. std.	0.005	0.001	0.001	0.001
	ESS	819	651	998	1015
b	Post. mean	0.441	0.386	0.322	0.196
	Post. std.	0.266	0.097	0.06	0.068
	ESS	580	533	852	781

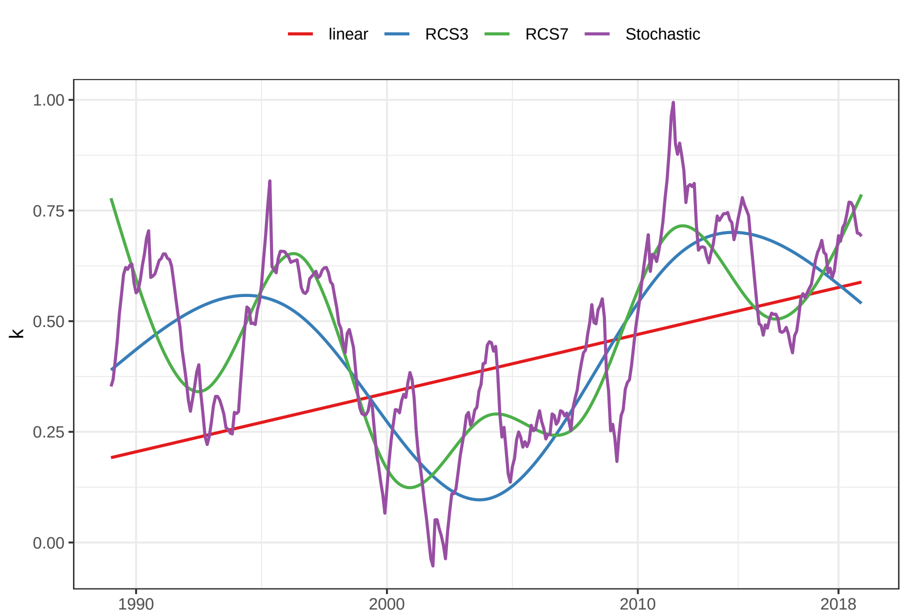
Cotton



Aluminium

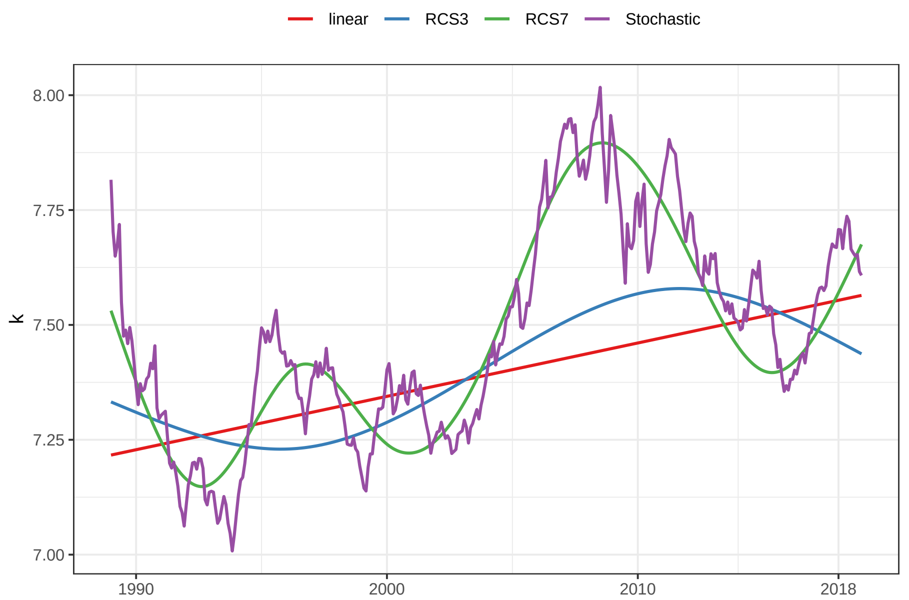


Cotton



linear
RCS3
RCS7
Stochastic
1.00
0.75
0.50
0.25
0.00
k
1990
2000
2010
2018

Aluminium


linear
RCS3
RCS7
Stochastic
k
8.00
7.75
7.50
7.25
7.00
1990
2000
2010
2018

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